16.2 Iterated Integrals

Suppose that f is a function of two variables that is integrable on the rectangle *R= [a, b] x [c, d].* We use the notation to mean that x is held fixed and *f(x, y)* is integrated with respect to y from *y=c* to *y=d.* This procedure is called **partial integration with respect to y.** Notice the similarity to the method for partial differentiation.

Now, is a number that depends on the value of *x*, so it defines a function of *x*:

.

If we now integrate *A(x*) with respect to *x* from *x=a* to *x=b*, we get

.

The integral on the right side of the equation is called an **iterated integral.** Typically, brackets are omitted to give

.

This method also works for partial integration with respect to .

**Note:** When integrating iterated integrals, always work the inside out. The inner integral corresponds to the innermost partial derivative. For example, look at the equation below:

As you can see, the innermost integral corresponds to the innermost differential

**Fubini’s Theorem** If f is continuous on the rectangle then

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In the special case where *f(x,y)* can be factored as the product of a function of *x* only and a function of *y* only, the double integral of *f* can be written in a particularly simple form:

where R=[a, b] x [c, d].

Chapter 16.7

Glossary:

The **Cartesian coordinate system** is a coordinate system in ℝ3 that represents the position of a point relative to the **origin** (the point at which all the coordinate axes intersect, denoted as ), as the distance of from the origin along each coordinate axis. That is to say, if is units away from the origin on the axis, units away from the origin on the axis, and units away from the origin on the axis, then

The **cylindrical coordinate system** is a coordinate system in ℝ3 that represents the position of a point relative to the origin as the combination of the radius of a circle equal in magnitude to the distance from the origin to point (denoted as ), the angle the line from the origin to makes with the axis (denoted as ), and the distance from the origin to along the axis (denoted as ), formally expressed as

If we want to switch from Cartesian coordinates to cylindrical, we can use the fact that , , and . If we want to switch from cylindrical to Cartesian, we can use the fact that , , and .

If a function over a type 1 region, where is polar region , then we can express the triple integral of over as

Using our definition for a double integral in polar coordinates, that is

We can form an iterated triple integral: